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A VISCOPLASTIC PLANE FRAME BEAM COLUMN ELEMENT FOR PROGRAM FEAP--ETC(U)
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A two-dimensional viscoplastic beam element was developed for the computer program FEAP. The element is suitable for small displacements or moderate rotations with negligible shear. The element is formulated in moment-thrust space which will improve modeling of concrete beams. The viscoplastic formulation provides time dependent effects such as the influence of the strain rate upon the yield strength.

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1. Introduction

This report describes a two dimensional viscoplastic plane frame beam element written for the computer program FEAP [1], and is a summary and extension of work presented by the first author in [2]. The element is suitable for the analysis of structural beam elements with rectangular or I shaped cross sections undergoing small displacement or moderate rotation deformations for which the influence of shear strain is negligible. The effects of both static and dynamic loadings may be considered.

The viscoplastic constitutive assumption is by definition a time dependent plasticity formulation, in which the rate of straining affects the ultimate yield strength. This is of particular interest in the analysis of structures subjected to high rate blast loadings. It is also possible to recover inviscid plasticity through a penalty approach by appropriate definition of the material properties.

1.1. Report Layout

Section Two describes the development of the weak form of the equations of equilibrium for the cases of small displacement and moderate rotation deformation. The constitutive theory and its adaptation to a beam formulation is presented in Section Three. Interpolations for stress resultants and centroidal axis displacements are introduced and used to assemble the governing algebraic equations for the element in Section Four. Numerical examples are presented in Section Five. Appendices I and II contain explicit descriptions of the component element matrices and of the Newmark time integration scheme used for solution of the nonlinear dynamics problem. A description of the element input data is found in Appendix III.

2. Weak Form Of The Equations Of Equilibrium

In accordance with the Bernoulli - Euler hypothesis that plane sections remain plane and do not rotate with respect to the centroidal axis, it is possible to define the state of stress in a beam using three stress resultants -- the axial force p, shear force v and bending moment m.

For a beam subjected to concentrated loads applied quasi-statically at the ends only, the momentum balance principles require that net force and moment on a differential segment must vanish. The positive conventions are taken as in Figure 1. In the case of small displacements, the equations for axial, transverse and rotational equilibrium take the form

$$p_{,x} = 0 \tag{1a}$$

$$v_{,x} = 0 \tag{1b}$$

$$m_{,x} = v \tag{1c}$$

where $(\ )_{,x}$ indicates differentiation with respect to variable x.

As no specific allowance is made for the effect of deformation due to shear, eqs. (1b) and (1c) may be combined to yield the following reduced set of equilibrium equations.

$$p_{,x} = 0 \tag{2a}$$

$$m_{,xx} = 0 \tag{2b}$$

These relations must be identically zero within the domain Ω^e of each element, and thus the weighted integral form of eqs.(2) must also vanish for arbitrary weight function $\mathbf{W}^t$. The **bold** type indicates a vectorial or matrix quantity, and superscript t indicates the transpose operation.

$$\int_{\Omega^e} \mathbf{W}^t \begin{Bmatrix} -p_{,x} \\ m_{,xx} \end{Bmatrix} d\Omega^e = 0 \tag{3}$$

Letting u represent the axial displacement and w the transverse displacement in a given beam element of length L, the weight function $\mathbf{W}^t$ may be chosen so that virtual work quantities result.

$$\mathbf{W}^t \equiv \{\ \delta u \quad \delta w\ \} \tag{4}$$

Substitution into eq.(3) followed by integration by parts results in the Galerkin form of the equations of equilibrium.

$$\int_0^L \{\ \delta u_{,x} \quad \delta w_{,xx}\ \} \begin{Bmatrix} p \\ m \end{Bmatrix} dx - \delta u p\Big|_0^L + \delta w m_{,x}\Big|_0^L - \delta w_{,x} m\Big|_0^L = 0 \tag{5}$$

The boundary terms in this equation form the natural boundary conditions for the problem, and hence may be replaced by the inner product of the virtual nodal displacements $\delta \mathbf{U}$ with the applied nodal loads $\mathbf{q}$.

It will be necessary to linearize this equation in order that an iterative Newton-Raphson solution scheme may be used. In anticipation of the rate dependence in the constitutive theory, a time stepping form of linearization is employed. Subscripts n represent the time step number, superscripts i the iteration counter within a step, and Δ an incremental quantity. Following the procedure employed in [2], the linearized form of the small displacement equilibrium equations is found to be

$$\int_0^L \{\delta u_{,x} \quad \delta w_{,xx}\} \begin{Bmatrix} p + \Delta p \\ m + \Delta m \end{Bmatrix}^i_{n+1} = \delta \mathbf{U}^t \mathbf{q}_{n+1} \tag{6}$$

For the case of moderate rotations, in which the transverse deflection w is of the order of the beam depth, the equilibrium equations are written [3]

$$p_{,x} = 0 \tag{7a}$$

$$v_{,x} = (pw_{,x})_{,x} \tag{7b}$$

$$m_{,x} = v \tag{7c}$$

As before, eqs. (7b) and (7c) are combined, yielding

$$p_{,x} = 0 \tag{8a}$$

$$m_{,xx} - (pw_{,x})_{,x} = 0 \tag{8b}$$

The Galerkin form for the moderate rotation equations of equilibrium is then obtained as in the previous case.

$$\int_0^L \left\{ (\delta u_{,x} + \delta w_{,x} w_{,x}) \quad \delta w_{,xx} \right\} \left\{ \begin{matrix} p \\ m \end{matrix} \right\} dx$$

$$- \delta up \Big|_0^L + \delta w m_{,x} \Big|_0^L - \delta w_{,x} m \Big|_0^L - \delta w p w_{,x} \Big|_0^L = 0 \tag{9}$$

Linearizing,

$$\int_0^L \left\{ (\delta u_{,x} + \delta w_{,x} w_{,x}) \quad \delta w_{,xx} \right\} \left\{ \begin{matrix} p + \Delta p \\ m + \Delta m \end{matrix} \right\}_{n+1}^{i}$$

$$+ \int_0^L \left\{ \delta w_{,x} p \Delta w_{,x} \right\} dx = \delta \mathbf{U}^t \mathbf{q}_{n+1} \tag{10}$$

The second integrand in eq.(10) contains a nonlinear term involving the product of axial force p with slope $w_{,x}$, which is known as the *geometric stiffness*, $\mathbf{K}_G$

3. Constitutive Theory

The viscoplastic constitutive theory[4] postulates that the total strain rate may be additively decomposed into the sum of elastic and inelastic components,

$$\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^i \tag{11}$$

with the elastic strain rate being related to the stress rate by the elastic compliance.

$$\dot{\epsilon}^e = \mathbf{C}^e \dot{\sigma} \tag{12}$$

A closed yield surface f is defined in stress (or stress resultant) space, such that for states of stress lying within the surface, the material response is purely elastic. The inelastic strain rate is then given by

$$\dot{\epsilon}^i = \gamma \langle \phi(f) \rangle \frac{\partial f}{\partial \sigma} \tag{13}$$

where

γ — *fluidity parameter*

$\phi(f)$ — *function chosen to represent the particular material*

$\langle \phi(f) \rangle$ — *operator such that* $\langle \phi(f) \rangle = 0, \quad f \leq 0$

$\langle \phi(f) \rangle = \phi(f), \quad f > 0$

It is apparent that the inelastic strain rate vector is directed parallel to the gradient of yield surface f, in accordance with Drucker's stability postulate[5]. In contrast to the classical plasticity theory, stress states outside of the yield surface are allowed, but only with non-zero rates of strain. This will result in a viscoplastic flow phenomenon. Similarly, if the displacement field is fixed by external constraints, the stresses will relax with time onto the yield surface, provided an equilibrium solution exists there.

The rate of viscoplastic flow or stress relaxation depends upon the stresses indirectly through the function $\phi(f)$, which for the current application is taken to be a power law[4].

$$\phi(f) \equiv f^{nnn} \tag{14}$$

The magnitude of inelastic strain rate is also affected by fluidity factor γ. For a given state of stress, if γ is doubled, the inelastic strain rate and hence inelastic strain increment are also doubled. By setting γ sufficiently large, it is possible to model inviscid plasticity. This forms the penalty approach alluded to in the introduction. The user is warned that γ may not be increased without bound, as indefinite stiffness matrices will result. Selection of reasonable values for γ is illustrated in the section on numerical results.

As a first step in adapting the constitutive theory to the beam element, it is necessary to define the yield surface f. Consistent with the restrictions stated in the introduction, it is sufficient to consider only the axial normal stress in determining f. The cross section is considered to be fully plastified when all fibers have yielded in tension or compression. Intermediate states are not included.

The stress distribution consists of two rectangular blocks, one in tension, the other in compression, and is parameterized by α, the distance from the centroidal axis of the section to the neutral axis of the stress distribution.(See Figure 2) When $\alpha = 0$, there is no axial force acting on the section, and when $\alpha = \pm h/2$ there is no bending moment. These represent the extrema of the axial force - bending moment interaction curve, or yield surface.

It is necessary to consider two separate cases, one in which the neutral axis lies in the web, and the other in which it lies in one of the flanges. This is facilitated [7] by splitting the stress blocks into symmetric and anti-symmetric parts with respect to the centroidal axis of the section. The symmetric part contributes only to axial force, and the anti-symmetric part only to bending moment.

Case 1: The neutral axis in web. $(\alpha \leq h/2 - t_f)$

The stress resultants p and m are first expressed in terms of the yield stress σ_y, the geometric properties of the section, and parameter α.

$$p = 2\alpha t_w \sigma_y \tag{15a}$$

$$m = m_p - t_w \sigma_y \alpha^2 \tag{15b}$$

Equations (15a) and (15b) are then combined to eliminate α, resulting in the following parabolic equation for a portion of the yield surface.

$$\left(\frac{m}{m_p}\right) + \frac{A p_p}{4 t_w m_p}\left(\frac{p}{p_p}\right)^2 - 1 = 0 \tag{16}$$

where

b	*section width*
h	*section height*
A	*section area*
t_f	*flange thickness*
t_w	*web thickness*

p_p *plastic force with zero moment*

m_p *plastic moment with zero force*

The expressions for plastic force and plastic moment are

$$p_p = A\sigma_y \tag{17}$$

$$m_p = \left[bt_f\left(h - t_f\right) + t_w\left(\frac{h}{2} - t_f\right)^2\right]\sigma_y \tag{18}$$

Case 2: The neutral axis in flange. $(\alpha > h/2 - t_f)$.

By a similar procedure we find that

$$p = p_p - \left(h - 2\alpha\right)b\sigma_y \tag{19a}$$

$$m = \left(h - 2\alpha\right)\left(\frac{h}{2} + \alpha\right)\frac{b\sigma_y}{2} \tag{19b}$$

which, upon combining to eliminate α, results in

$$\left(\frac{m}{m_p}\right) - \frac{Ap_p}{4bm_p}\left(1 - \frac{p}{p_p}\right)\left(\frac{2hb}{A} - 1 + \left(\frac{p}{p_p}\right)\right) = 0 \tag{20}$$

Together, equations (16) and (20) define the yield surface for positive values of bending moment. If these functions are plotted using the dimensionless variables

$$\left(\frac{p}{p_p}\right) \text{ and } \left(\frac{m}{m_p}\right)$$

the function is symmetric about the normalized moment axis. When the sign of the stresses in the assumed distribution is reversed, the yield functions for negative moment are generated. Due to the double symmetry of the cross sections under consideration, the negative moment yield functions are simply the reflection of the positive moment functions about the normalized force axis. If the section is rectangular, the neutral axis is always in the web, and the yield function reduces to the familiar parabolic function.

$$\left(\frac{p}{p_p}\right)^2 \pm \left(\frac{m}{m_p}\right) - 1 = 0 \tag{21}$$

When the positive moment yield function f^+ is plotted together with the negative moment yield function f^-, there is a discontinuity in the slope along the normalized force axis, forming what is called a corner condition. To obtain a continuous variation of the inelastic strain rate vector as the corners are approached, a summation of the two yield surfaces will be adopted[6]. The yield functions are not truncated at their intersections, but extended indefinitely. Three types of region are thus formed; one in which neither function is active, the second in which only one function is active, and the third in which both are active. The resultant strain rate vector is taken as the vector sum of the contributions from all active yield functions. This provides a continuous variation around the corner region, and also gives a vector parallel to the force axis when there is zero bending moment.

It is necessary to discretize the constitutive rate equation in time, and to linearize it with respect to the primary dependent variables, ϵ and σ. Following the procedure employed in [2], the rate terms are replaced with an Euler backward difference, leading to

$$\phi^i_{n+1} = \left(\epsilon^i_{n+1} - \epsilon_n\right) - C^e\left(\sigma^i_{n+1} - \sigma_n\right) + \gamma\Delta t\left\{<\phi(f^+)>\frac{\partial f^+}{\partial \sigma} + <\phi(f^-)>\frac{\partial f^-}{\partial \sigma}\right\}^i_{n+1} \tag{22}$$

The elastic compliance is

$$\mathbf{C}^{e} = \begin{bmatrix} \dfrac{1}{EA} & 0 \\ 0 & \dfrac{1}{EI} \end{bmatrix}$$

The vector of stresses is

$$\sigma = \begin{Bmatrix} p \\ m \end{Bmatrix}$$

The vector of strains for small displacements is

$$\epsilon = \begin{Bmatrix} u_{,x} \\ w_{,xx} \end{Bmatrix}$$

The vector of strains for moderate rotations is

$$\epsilon = \begin{Bmatrix} u_{,x} + \dfrac{w_{,x}^{2}}{2} \\ w_{,xx} \end{Bmatrix}$$

Taking the linear part of eq.(22) and setting it equal to zero defines the tangent constitutive law. The tangent compliance, $\mathbf{C}_T$, will contain contributions from each of the yield functions if they are active. For the specific form, see [2].

$$L\left[\psi_{n+1}^{i+1}\right] = \psi_{n+1}^{i} + \Delta\epsilon_{n+1}^{i} - \mathbf{C}_T \Delta\sigma_{n+1}^{i} = 0 \tag{23}$$

Weighting this equation with a virtual stress field and integrating over Ω^e defines the Galerkin form.

$$\int_0^L \mathbf{G}^t \left[\Delta\epsilon_{n+1}^{i} - \mathbf{C}_T \Delta\sigma_{n+1}^{i} + \psi_{n+1}^{i}\right] dx = 0 \tag{24a}$$

$$\mathbf{G}_T \equiv \begin{Bmatrix} \delta p & \delta m \end{Bmatrix} \tag{24b}$$

The linearization of the moderate rotation strain vector,

$$\Delta\epsilon_{n+1}^{i} = \begin{Bmatrix} \Delta u_{,x} + w_{,x}\Delta w_{,x} \\ \Delta w_{,xx} \end{Bmatrix}_{n+1}^{i} \tag{25}$$

results in virtual work terms which are conjugate to those appearing in the weak form of equilibrium. Hence, the symmetry of the linearized equations is preserved for moderate rotations as well as small displacements.

The derivatives of f^{+} and f^{-} required in eqs.(22) and (23) are conveniently evaluated using the chain rule.

$$\frac{\partial f}{\partial p} = \frac{1}{p_p}\frac{\partial f}{\partial (p/p_p)} \tag{26}$$

$$\frac{\partial f}{\partial m} = \frac{1}{m_p}\frac{\partial f}{\partial (m/m_p)} \tag{26b}$$

To keep the equations dimensionally consistent, it is necessary to replace fluidity γ by γp_p.

4. Finite Element Interpolations

In order that the integrals in the Galerkin forms of the equilibrium and constitutive equations may be evaluated, it is necessary to specify interpolation functions for the stress resultants and the centroidal axis strains.

The displaced shape of the element will be expressed in terms of C^0 linear shape functions for axial displacement $u(x)$, and C^1 Hermite polynomials for transverse displacement $w(x)$.

$$\begin{Bmatrix} u(x) \\ w(x) \end{Bmatrix} = \begin{bmatrix} N_1(x) & 0 & 0 & N_2(x) & 0 & 0 \\ 0 & H_1(x) & H_2(x) & 0 & H_3(x) & H_4(x) \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{Bmatrix} \tag{26}$$

which in matrix notation becomes

$$\mathbf{u}(x) = \mathbf{N}(x)\mathbf{U} \tag{27}$$

Strain interpolation relations are obtained by appropriately differentiating the displacement shape functions.

For small displacements, the incremental strain-displacement operator is linear, and results in

$$\mathbf{B}(x)\Delta \mathbf{U}^{i}_{n+1} = \begin{Bmatrix} \Delta u_{,x} \\ \Delta w_{,xx} \end{Bmatrix} \tag{28}$$

For moderate rotations, however, the operator depends upon the current transverse displacement field, $w(x)$.

$$\mathbf{B}(x,w)\Delta U^{i}_{n+1} = \begin{Bmatrix} \Delta u_{,x} + w_{,x}\Delta w_{,x} \\ \Delta w_{,xx} \end{Bmatrix} \tag{29}$$

The stress resultants in the element will be interpolated using piecewise continuous functions, which will allow for discontinuities in the stress field between elements.

$$\begin{Bmatrix} p(x) \\ m(x) \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1-\dfrac{x}{L} & \dfrac{x}{L} \end{bmatrix} \begin{Bmatrix} p \\ m_1 \\ m_2 \end{Bmatrix} \tag{30}$$

or

$$\sigma^{i}_{n+1}(x) \equiv \mathbf{S}(x)\hat{\sigma}^{i}_{n+1} \tag{31}$$

Substituting these interpolations into the linearized Galerkin forms for the small displacements equations leads to

$$\delta \mathbf{U}^t \int_0^L \mathbf{B}^t(x)\mathbf{S}(x)\,dx\,\Delta\hat{\sigma}^{i}_{n+1} = \delta \mathbf{U}^t \left\{ \mathbf{q}_{n+1} - \int_0^L \mathbf{B}^t(x)\mathbf{S}(x)\,dx\,\hat{\sigma}^{i}_{n+1} \right\} \tag{32}$$

and

$$\delta\hat{\sigma}^t \left\{ \int_0^L \mathbf{S}^t(x)\mathbf{B}^t(x)\,dx\,\Delta U^{i}_{n+1} - \int_0^L \mathbf{S}^t(x)\mathbf{C}_T\mathbf{S}(x)\,dx\,\Delta\hat{\sigma}^{i}_{n+1} \right\} = \delta\hat{\sigma}^t \int_0^L \mathbf{S}^t(x)\psi^{i}_{n+1}\,dx \tag{33}$$

Evaluation of the integrals by three point Gauss-Lobatto numerical quadrature results in the following mixed system of equations

$$\begin{bmatrix} 0 & \mathbf{T}' \\ \mathbf{T} & -\mathbf{Q}_T \end{bmatrix} \begin{Bmatrix} \Delta \mathbf{U}^i_{n+1} \\ \Delta \hat{\sigma}^i_{n+1} \end{Bmatrix} = \begin{Bmatrix} \mathbf{q}_{n+1} - \mathbf{T}'\hat{\sigma}^i_{n+1} \\ \Psi^i_{n+1} \end{Bmatrix} \tag{34}$$

The stress interpolation parameters $\hat{\sigma}$ are by definition piecewise continuous from element to element. Hence, the mixed system may be statically condensed to yield a generalized displacement model.

$$\mathbf{T}'\mathbf{Q}_T^{-1}\mathbf{T}\ \Delta \mathbf{U}^i_{n+1} = \mathbf{q}_{n+1} - \mathbf{T}'\hat{\sigma}^i_{n+1} + \mathbf{T}'\mathbf{Q}_T^{-1}\Psi^i_{n+1} \equiv \mathbf{R}^i_{n+1} \tag{35a}$$

$$\Delta \hat{\sigma}^i_{n+1} = -\mathbf{Q}_T^{-1}\,(\Psi^i_{n+1} - \mathbf{T}\Delta \mathbf{U}^i_{n+1}) \tag{35b}$$

In the moderate rotation case, the nonlinear incremental strain displacement matrix $\mathbf{B}(x,w)$ must be used, and the geometric stiffness $\mathbf{K}_G$ is added to the weak form of equilibrium.

$$\begin{bmatrix} \mathbf{K}_G & \mathbf{T}'(w) \\ \mathbf{T}(w) & -\mathbf{Q}_T \end{bmatrix} \begin{Bmatrix} \Delta \mathbf{U}^i_{n+1} \\ \Delta \hat{\sigma}^i_{n+1} \end{Bmatrix} = \begin{Bmatrix} \mathbf{q}_{n+1} - \mathbf{T}'(w)\hat{\sigma}^i_{n+1} \\ \Psi^i_{n+1} \end{Bmatrix} \tag{36}$$

Statically condensing,

$$\left[\mathbf{K}_G + \mathbf{T}'(w)\mathbf{Q}_T^{-1}\mathbf{T}(w)\right]\Delta \mathbf{U}^i_{n+1} = \mathbf{q}_{n+1} - \mathbf{T}'(w)\hat{\sigma}^i_{n+1} + \mathbf{T}'(w)\mathbf{Q}_T^{-1}\Psi^i_{n+1} \equiv \mathbf{R}^i_{n+1} \tag{37a}$$

$$\Delta \hat{\sigma}^i_{n+1} = -\mathbf{Q}_T^{-1}\,(\Psi^i_{n+1} - \mathbf{T}(w)\Delta \mathbf{U}^i_{n+1}) \tag{37b}$$

5. Numerical Results

To illustrate the selection of material properties for the viscoplastic model, two sample problems involving the axial extension of a one element bar were undertaken.

In the first example, a stress relaxation test, the bar was given an instantaneous axial extension and the axial force plotted versus time for various values of the fluidity parameter γ. In all cases, the elastic solution was taken as the initial condition. The results of this test are illustrated in Figure 3. It is apparent that as γ is increased, the time required for the stresses to relax onto the yield surface decreases in an exponential manner. Since the constitutive equation is of necessity discretized in time, the limiting state of inviscid plasticity may be considered to have been attained when the time required for relaxation onto the yield surface is less than one time step Δt. The value of γ corresponding to this limiting state depends upon the material properties as well as on the initial stress excursion outside of the yield surface. The user is warned that values of γ significantly larger than this limiting value may result in physically improbable solutions, such as axial contraction of an element when subjected to a tensile force above the yield load. Hence, it is imperative that proper care be taken in specification of the material properties.

The second example illustrates the effect of strain rate on the yield stress. The end of the bar was extended axially at constant velocity, and axial stress plotted versus axial strain for various velocities. The value of γ was held constant throughout. Results of this example are plotted in Figure 4. When the strain rate is increased, the effective yield stress is also increased. In the limiting case of vanishingly small strain rate, the inviscid plastic response is recovered.

The inelastic material response is also dependent upon the value of the exponent used in the power law $\phi(f)$. A similar set of examples should also be undertaken by the user to examine response sensitivity to this parameter. It has been found by the authors that convergence difficulties occur if the exponent *nnn* is set equal to 1 whenever bending moment gradients are present in the equilibrium solution. As a practical consideration therefore, the value of *nnn* should always be set equal to or greater than 2.

To illustrate the use of the element in a dynamic analysis, the case of blast loading on a simply supported beam was considered. The loading was idealized as a uniformly distributed triangular pulse, with initial amplitude equal to 2.5 times the static collapse load, and duration 0.5 times the fundamental period. Due to the symmetry of the structure and loading, only one half of the span was modeled, using ten elements of equal length. Small displacement kinematics were assumed. The value of γ was set to model inviscid plasticity.

Time history plots were made of the midspan deflection and bending moment (Figures 5 & 6). Plots were also made of the distribution of bending moment along the half span at every five time steps. (Figures 7,8 & 9) It is apparent from the time history plots that the response is primarily in the first mode of the structure. The period of constant bending moment (yielding phase) is accompanied by a parabolic displacement time history. After sufficient energy is dissipated through inelastic deformation, the structure responds in simple harmonic motion at the fundamental frequency about a residual plastic deformation. The 'noise' which appears in the moment history and distributions is a consequence of the inertia force contribution from accelerations in the higher modes.

The overall response of the beam element to blast loading is most satisfactory, and compares favorably with a single degree of freedom approximation given in [8] .

References

[1] Zienkiewicz,O.C., *The Finite Element Method*, 3rd ed.,McGraw-Hill,London (1977).

[2] Slater,J.H.,"Mixed-Model Finite Element Analysis of Inelastic Plane Frames", CE299 Report No. 750, Department of Civil Engineering, University of California, Berkeley (October 1980).

[3] Washizu,K., *Variational Methods in Elasticity and Plasticity*, 2nd ed., Pergamon Press (1974).

[4] Perzyna,P., "Fundamental Problems in Viscoplasticity", *Advances in Applied Mechanics*, Vol.9,pp.243-377(1966).

[5] Hodge,P.G., *Plastic Analysis of Structures*, McGraw-Hill (1959).

[6] Mroz,Z. and Sharma,K.G., "Finite Element Applications of Viscoplasticity with Singular Yield Surfaces", *I.J.N.M.E.*, vol.15,no.3,pp.411-436(1980).

[7] Horne,M.R., *Plastic Theory of Structures*, The MIT Press (1971).

[8] Biggs,J.M., *Introduction to Structural Dynamics*, McGraw-Hill (1964).

[9] Hughes,T.J.R., Pister,K.S. and Taylor,R.L., "Implicit-Explicit Finite Elements In Non-linear Transient Analysis", *Computer Methods in Applied Mechanics and Engineering*, vol.17/18,pp.159-182(1979).

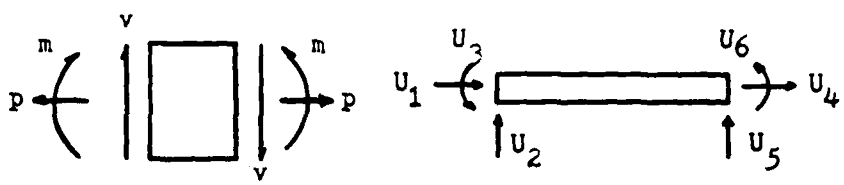

Figure 1 - Sign Conventions

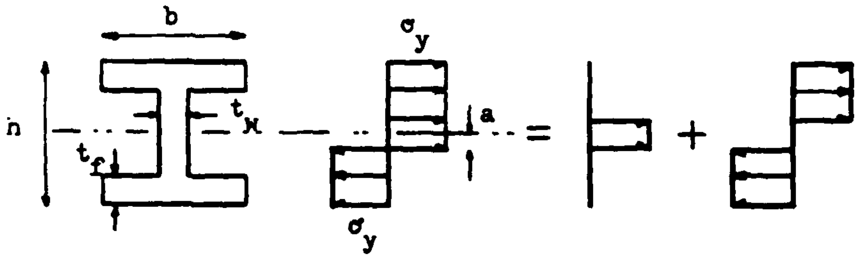

Figure 2 - Fully Plastic Stress Distribution

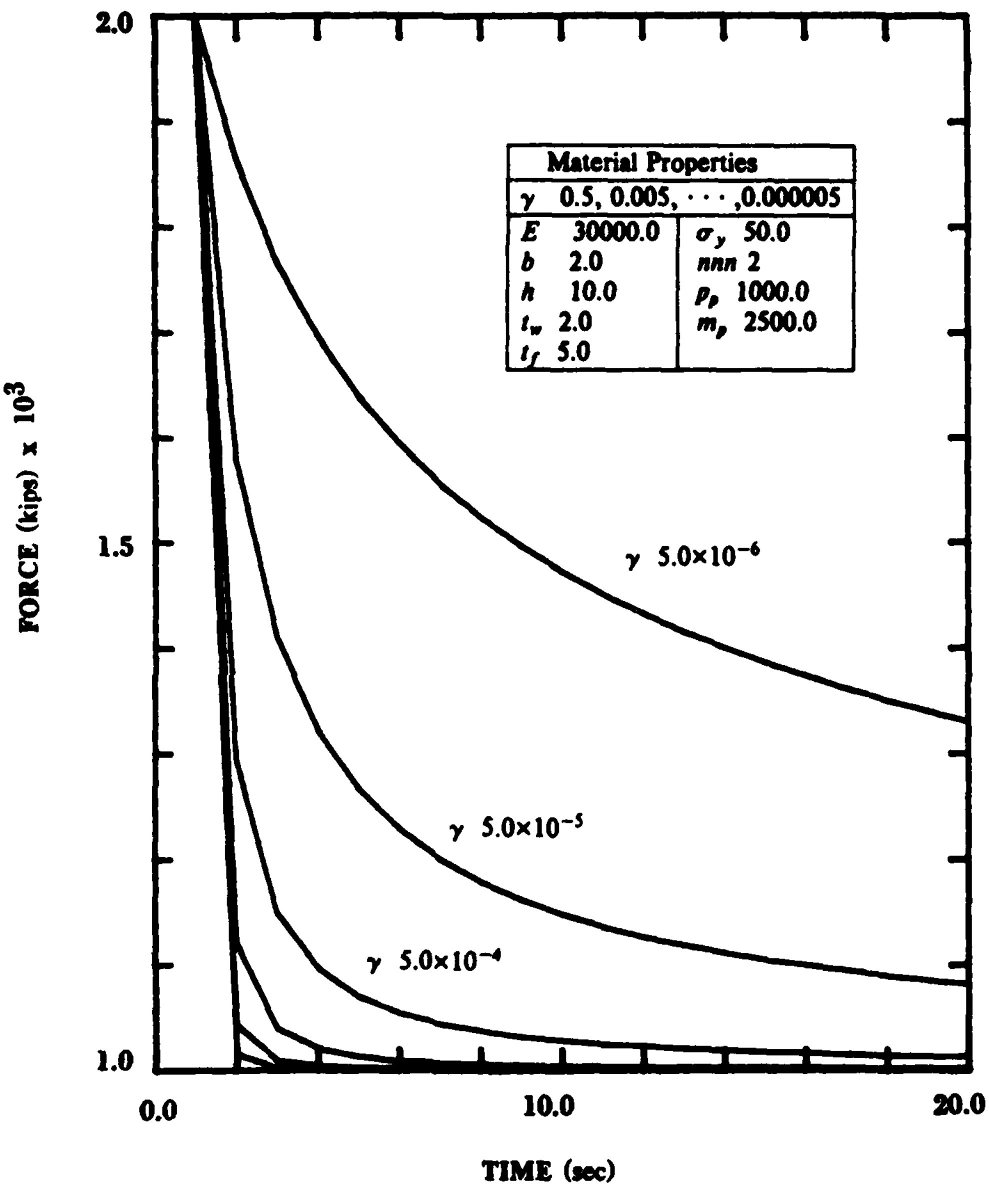

Figure 3 - Axial Force Relaxation

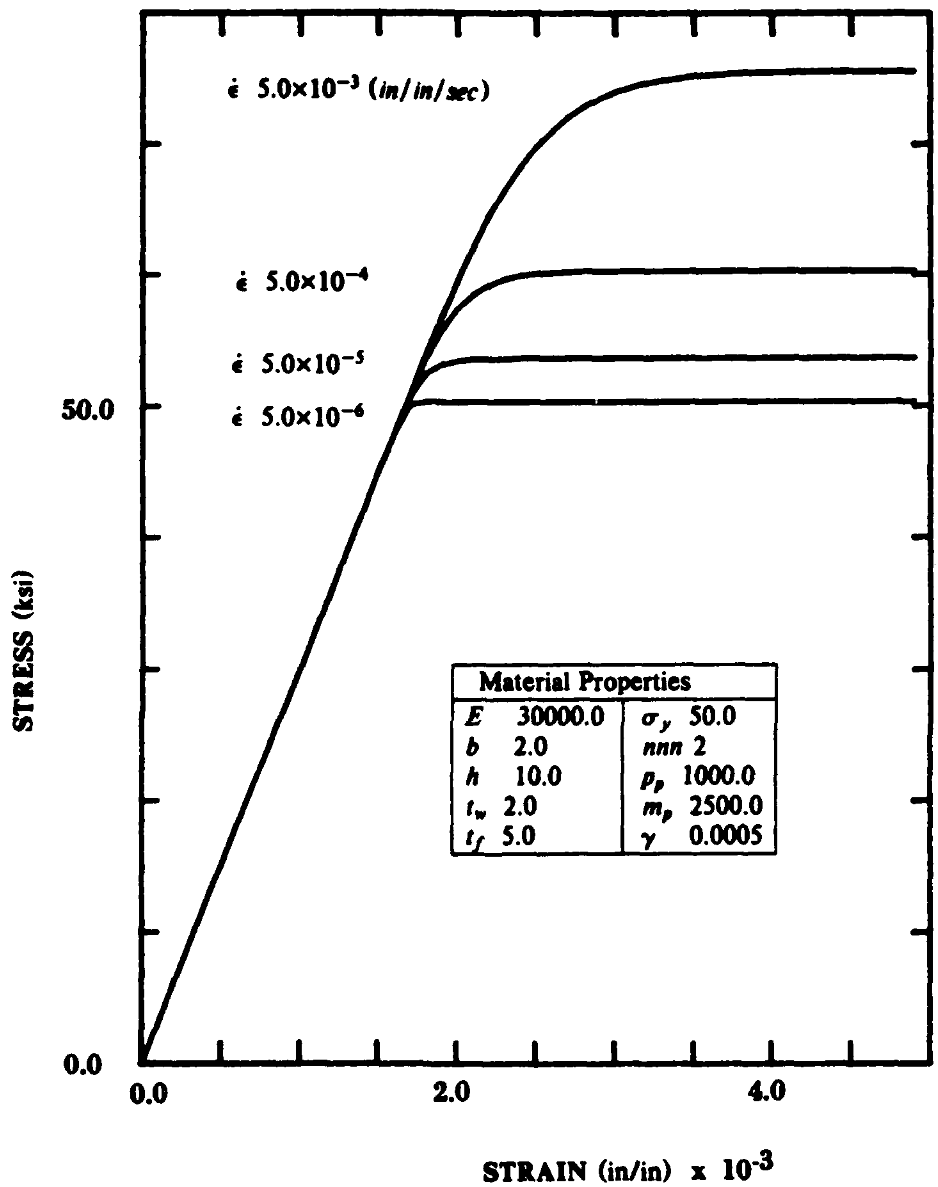

Figure 4 - Strain Rate Sensitivity of Yield Stress

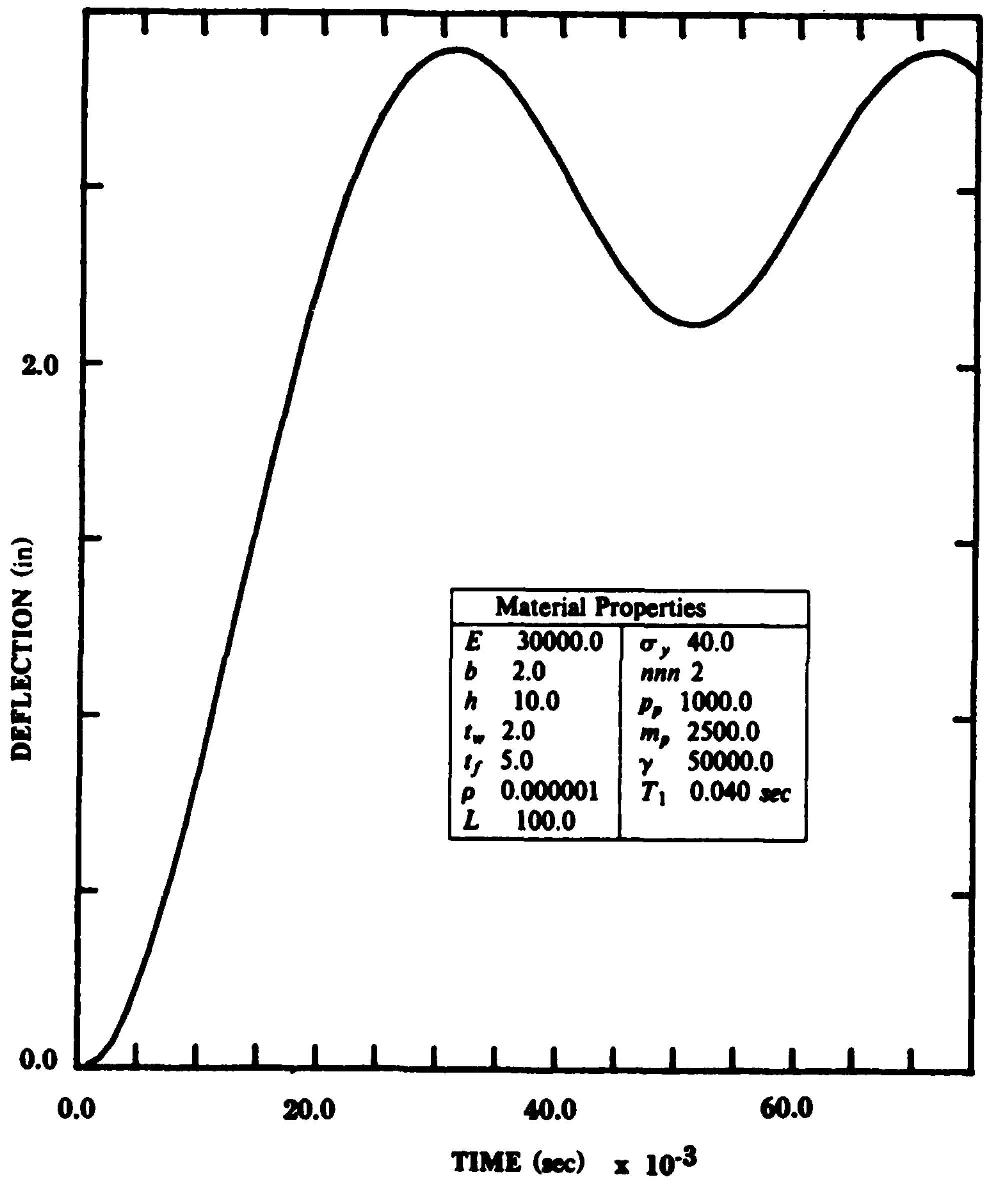

Figure 5 - Time History of Midspan Deflection

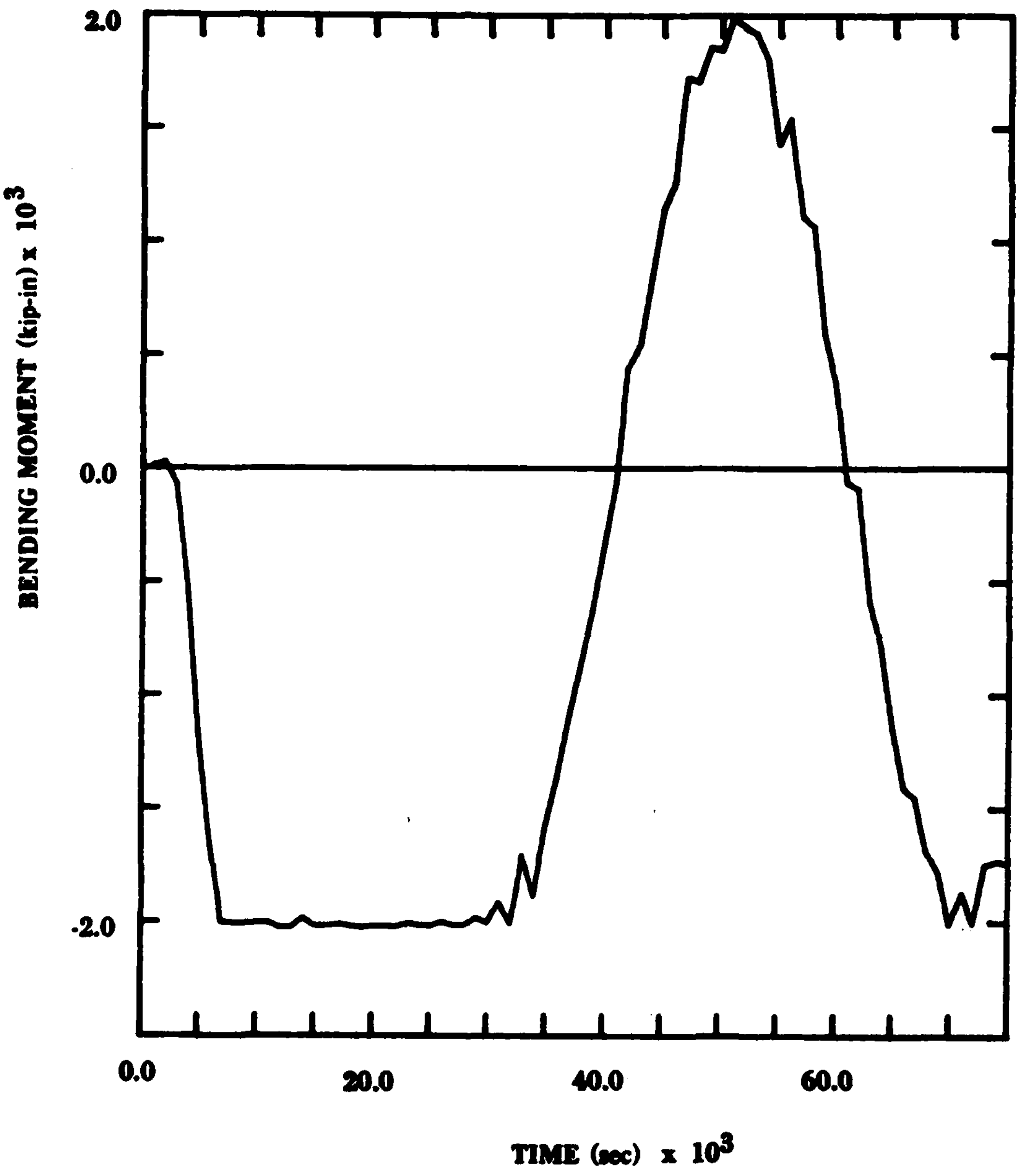

Figure 6 - Time History of Midspan Bending Moment

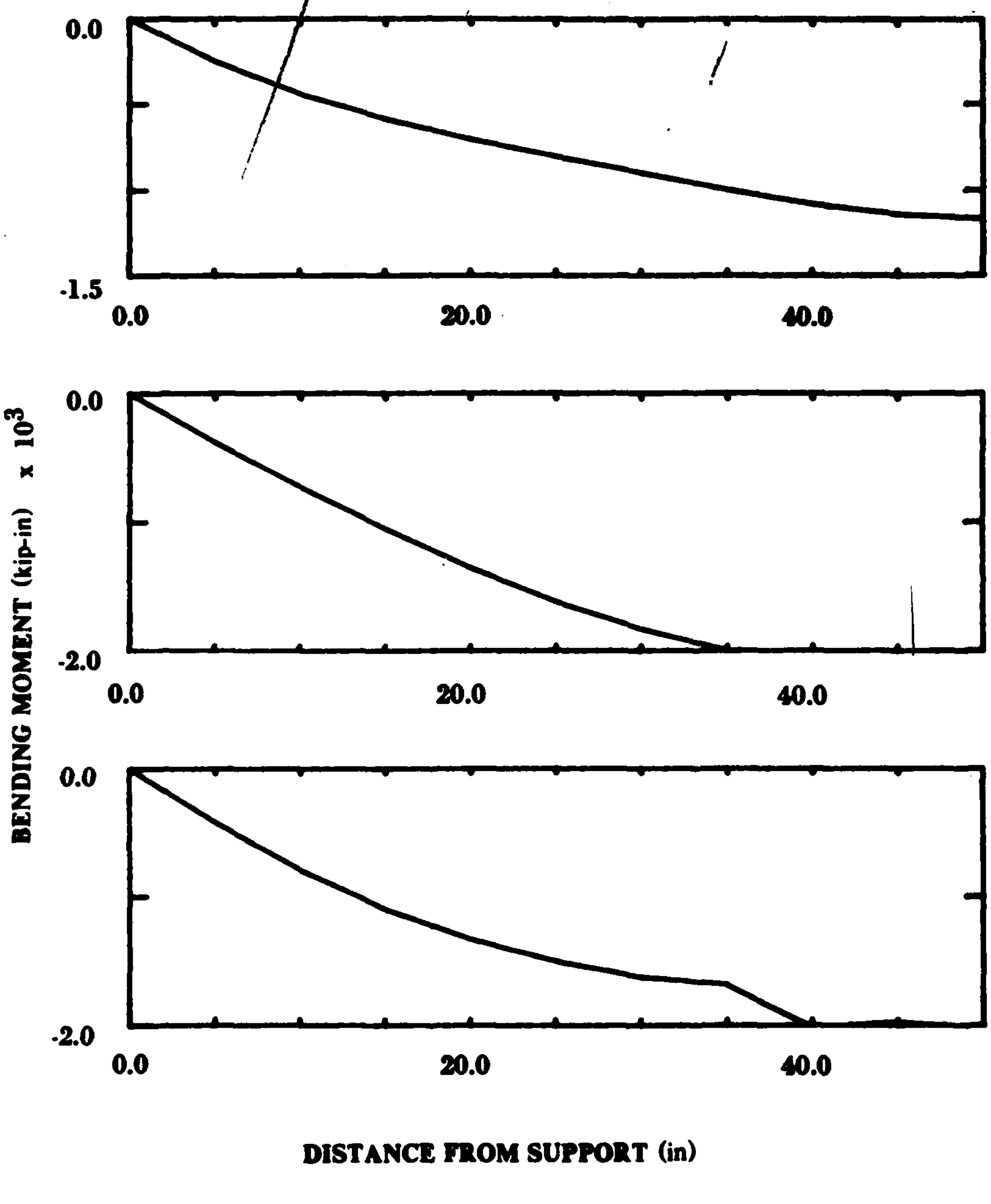

Figure 7 - Bending Moment At Time 0.005, 0.010 & 0.015

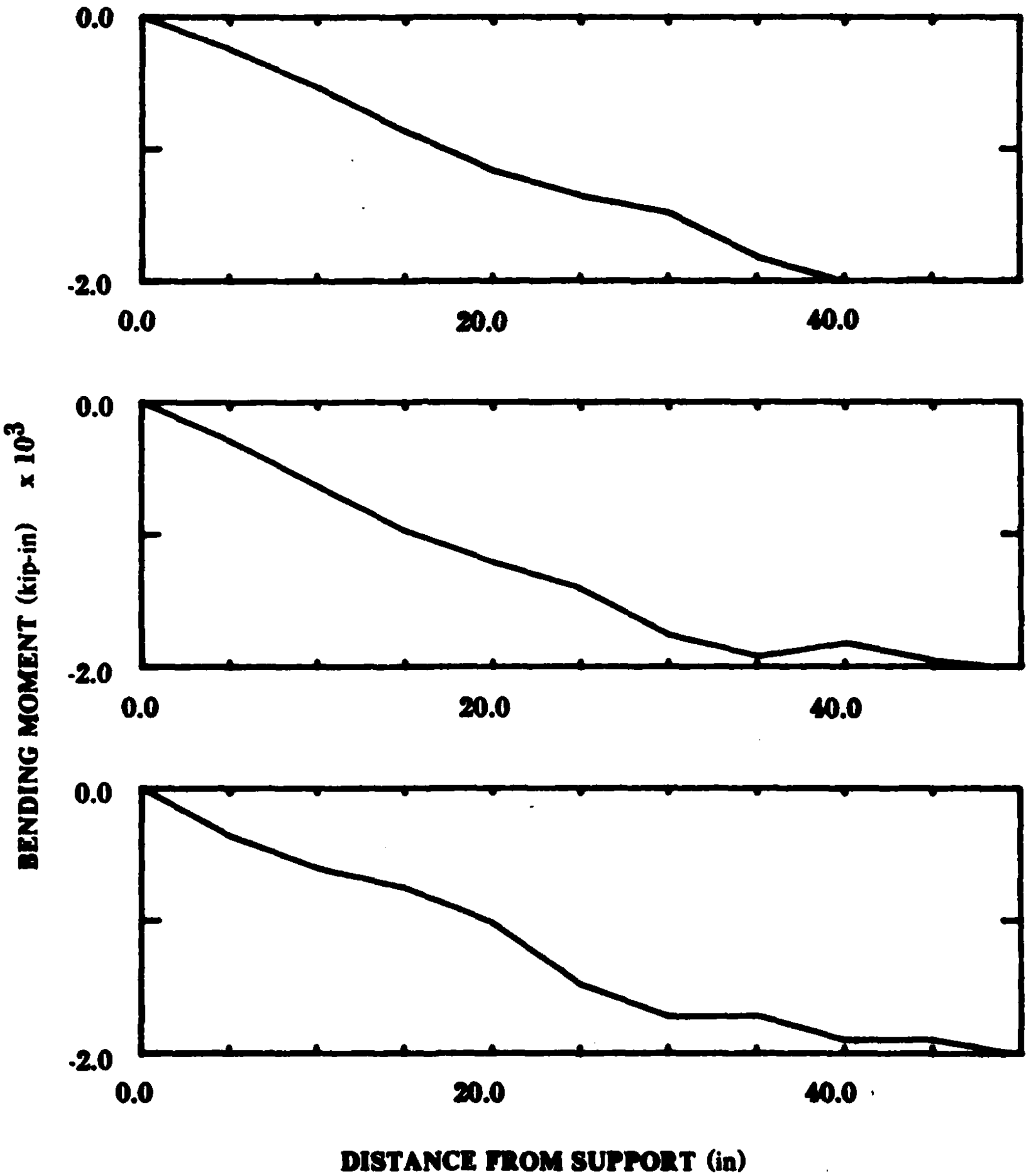

Figure 8 - Bending Moment At Time 0.020, 0.025 & 0.030

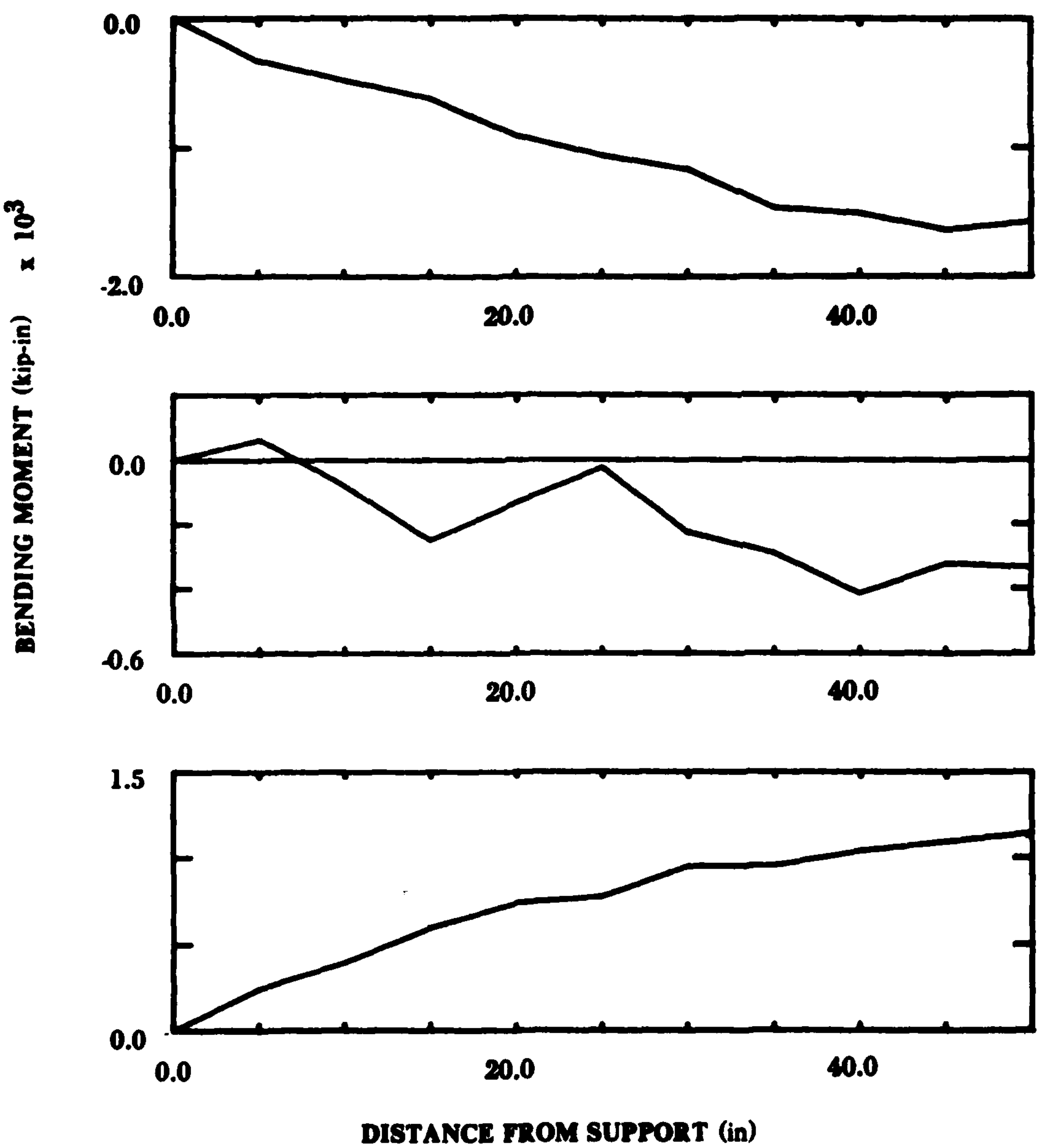

Figure 9 - Bending Moment At Time 0.035, 0.040 & 0.045

Appendix I - Component Matrices For Element Equations

The beam element equations are given in general form by eqs.(34) and (36), and contain three typical submatrices which result from the weak form of the equilibrium and constitutive equations. They are the geometric stiffness $\mathbf{K}_G$, the flexibility $\mathbf{Q}$, and the transformation matrix $\mathbf{T}$. Substitution from the specified interpolations allows these matrices to be integrated exactly, resulting in

The geometric stiffness (non - zero terms only):

$$\mathbf{K}_G = \frac{P}{30L} \times \begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix}$$

The flexibility (elastic case only):

$$\mathbf{Q}_T = \begin{bmatrix} \frac{L}{EA} & 0 & 0 \\ 0 & \frac{L}{3EI} & \frac{L}{6EI} \\ 0 & \frac{L}{6EI} & \frac{L}{3EI} \end{bmatrix}$$

The transformation matrix (small displacements):

$$\mathbf{T} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{-1}{L} & -1 & 0 & \frac{1}{L} & 0 \\ 0 & \frac{1}{L} & 0 & 0 & \frac{-1}{L} & 1 \end{bmatrix}$$

The transformation matrix (moderate rotations):

$$\mathbf{T}(w) = \begin{bmatrix} -1 & a & b & 1 & c & d \\ 0 & \frac{-1}{L} & -1 & 0 & \frac{1}{L} & 0 \\ 0 & \frac{1}{L} & 0 & 0 & \frac{-1}{L} & 1 \end{bmatrix}$$

The terms a, b, c, and d appearing in the moderate rotation case are the contraction of the geometric stiffness $\mathbf{K}_G$ with the current transverse displacement field parameters $\mathbf{w}$.

Appendix II - Time Integration By The Newmark Method

The incremental dynamic equations of equilibrium for a discrete system are written in matrix form as

$$\mathbf{M}\Delta\mathbf{A}^{i}_{n+1} + \mathbf{K}_T\Delta\mathbf{U}^{i}_{n+1} = \mathbf{R}^{i}_{n+1} - \mathbf{M}\mathbf{A}^{i}_{n+1}$$

where $\mathbf{M}$ is a diagonal lumped mass matrix[8] , $\mathbf{A}$ the vector of nodal accelerations, $\mathbf{K}_T$ the tangent stiffness matrix, $\mathbf{U}$ the nodal displacement vector and $\mathbf{R}$ the externally applied loads less internal resisting forces, all evaluated at iteration i in time step $n+1$.

These equations are coveniently solved using the Newmark formulas in a formulation having displacement as primary dependent variable[9]. Accordingly,

$$\hat{\mathbf{U}}_{n+1} = \mathbf{U}_n + \Delta t\mathbf{V}_n + \frac{\Delta t^2}{2}(1 - 2\beta)\mathbf{A}_n$$

$$\hat{\mathbf{V}}_{n+1} = \mathbf{V}_n + \Delta t(1 - \gamma)\mathbf{A}_n$$

The steps for a typical iteration are as follows.

Predictor phase.

$$\mathbf{U}^{i}_{n+1} = \hat{\mathbf{U}}^{i}_{n+1}$$

$$\mathbf{V}^{i}_{n+1} = \hat{\mathbf{V}}_{n+1}$$

$$\mathbf{A}^{i}_{n+1} = 0$$

Out of balance forces.

$$\Delta\mathbf{R} = \mathbf{R}^{i}_{n+1} - \mathbf{M}\mathbf{A}^{i}_{n+1} - \mathbf{K}_T\mathbf{U}^{i}_{n+1}$$

Effective stiffness formulation.

$$\mathbf{K}^* = \frac{1}{\Delta t^2\beta}\mathbf{M} + \mathbf{K}_T$$

$$\mathbf{K}^*\Delta\mathbf{U} = \Delta R$$

Corrector phase.

$$\mathbf{U}^{i+1}_{n+1} = \mathbf{U}^{i}_{n+1} + \Delta\mathbf{U}$$

$$\mathbf{A}^{i+1}_{n+1} = \left(\mathbf{U}^{i+1}_{n+1} - \hat{\mathbf{U}}_{n+1}\right)/(\Delta t^2\beta)$$

$$\mathbf{V}^{i+1}_{n+1} = \hat{\mathbf{V}}_{n+1} + \Delta t\gamma\mathbf{A}^{i+1}_{n+1}$$

Iterations are performed until a satisfactory convergence tolerance is met. The time step is then advanced and the procedure repeated.

Appendix III - Beam Element Input Data

This appendix describes the input data necessary to use the plane frame viscoplastic beam element described in the preceding report. General input data required for use of program FEAP is outlined in chapter 24 of [1], and will not be covered here. However, a copy of the input used for the blast load analysis example appears at the end of this section.

The following information will be required on Card 2 of the general input data:

Dimension of coordinate space - 2
Degrees of freedom per node - 3
Nodes per element - 2
Added degrees of freedom - 0

The following information will be required during material property specification following the *mate* macro command (3 cards):

Card 1 - format (2i10)

Field 1 - material set number as specified in *elem* macro
Field 2 - 1 (for element elmt01)

Card 2 - format (5x,a5,7f10.0)

Field 1 - *small* or *large* for small or moderate deformation kinematics
Field 2 - Young's modulus E (ksi)
Field 3 - section width b (in)
Field 4 - section depth h (in)
Field 5 - web thickness t_w (in)
Field 6 - flange thickness t_f (in)
Field 7 - yield stress in tension σ_y (ksi)
Field 8 - fluidity parameter γ (1/sec)

Card 3 - (5f10.0)

Field 1 - exponent *nnn* with minimum value of 2, used as integer
Field 2 - mass density ρ (kip-sec**2/in**4)
Field 3 - shift of yield surface origin in normalized moment direction
Field 4 - shift of yield surface origin in normalized force direction
Field 5 - tolerance on yield criterion *ftol* (default 0.00001)

The shifts of the yield surface origin allow the user to move the yield surface along the two normalized stress axes. The tolerance on the yield criterion allows a stress state within *ftol* of the yield surface to be considered elastic.

Input data for blast loading example :

```
feap  Sample input deck for simply supported beam
       11     10      1      2      3      2      0
coor
        1      1    0.0    0.0    0.0
       11      0   50.0    0.0    0.0
(blank card)
elem
        1      1      1      2      1
(blank card)
boun
        1      0      1      1      0
       11      0      0      0      1
(blank card)
forc
        2      1      0    1.0    0.0    0.0
       10      0      0    1.0    0.0    0.0
       11      0      0    0.5    0.0    0.0
(blank card)
mate
        1      1
    small  30000.0    2.0   10.0    5.0   40.0  50000.0
      2.0  0.00001    0.0    0.0    0.0
(blank card)
end
macr
prop         1
tol    0.00001
dt       0.001
beta      0.25    0.5
lmas
loop        76
time
loop        12
tang
form
```

```
solv
next
disp
stre
next
end
      1     0     0.0     0.02     20.0   -1000.0     0.0
stop
```

ME

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